Absolute angle estimation by means of an IMU in a manipulator with electrohydraulic servo drives

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Abstract: The problem of estimation of the manipulator’s arm angle using the inertial measurement unit (IMU) is discussed. This unit was attached to the arm allowing identification of the arm’s angle relatively to the global coordinate system. The manipulator was also equipped with two incremental encoders. Results of conducted experiments allowed to compare the pitch angle of the robot arm, estimated from the IMU and calculated from the encoder unit. In the study the influence of the IMU sensor position on the quality of estimates was verified. Parameters of the estimation algorithm have been also checked. Finally, the selected estimation algorithm was verified during the operation, where manipulator moved at various speeds and angles. Aim of this study was to test the angle estimation method using an IMU in the mechanical system with hydraulic drives.

Keywords: Kalman Filter, IMU, electrohydraulic manipulator, encoder

Measurement of the angle of the manipulator’s arm is a task normally performed by the encoders, most often incremental. They allow to determine the tilt angle directly by counting the pulses in each robot’s joint. This method enables accurate and robust closed-loop control. Encoder sensors, however, are expensive, generally more expensive than MEMS (Micro Electro-Mechanical Systems) sensors which tend to become even cheaper. The use of MEMS sensors would allow significant reduction of manufacturing costs of manipulators. In the literature attempts to introduce estimating systems for the position and orientation of the manipulator’s joints based on accelerometers [1] and [2] can be seen ever more often.

Additionally, some works [3] introduce sophisticated methods that estimate the orientation and position of the manipulator’s arm using accelerometers and gyroscopes. In this paper the authors attempt to introduce a simple system for estimating the tilt angle of the first arm of the robot using an IMU. For this purpose, hydraulic manipulator was used as a reference. It allows to compare the angle estimated form the IMU with the one calculated from the standard encoder. Authors decided to choose a hydraulic manipulator because of the possibility to set the position of the arm with motion of high dynamics and with disturbances characteristic for hydraulic systems.

1. The manipulator

Hydraulic manipulator, on which the research was conducted, is a system with three degrees of freedom, reminiscent of the construction of the lower limb without foot (fig. 1).

Fig. 1. Dimensions of the manipulator

Rys. 1. Wymiary manipulatora

It is a system equipped with three hydraulic pistons and servo-electric control. Master system imposes the robot’s TCP (Tool Center Point) position by actuation of the hydraulic valve and simultaneous analysis of the position of the arm. Main feature of the above mentioned system is high dynamics of manipulator’s movements. The manipulator provides a very good reference signal of changes in the position of the arm used in evaluation of the estimating algorithm.

2. Angle estimation

Angle estimation algorithms are based on MEMS sensors. The group of such sensors includes gyroscopes and accelerometers. These are inexpensive sensors, in which the fusion of information allows to obtain very good estimates of the measured physical quantity, assuming that setting, the estimation algorithm and the quality of the sensors are well suited to the application.

The problem with this type of system is an error which depends on the dynamics of the object on which the sensors are mounted. It is difficult to talk about the optimal settings for the estimator. Depending on the application in which the system works, suitable parameters should be set. A key issue is the algorithm for estimating a given quantity.

2.1. Estimation filters

Currently, different angle estimation algorithms are used in applications with IMU. Frequently Kalman filter is
exploited as the minimum variance estimator. It works in the following two stages:
1) Prediction of the estimated state, based on a process model or measurement.
2) Correction of the estimated state, based on a measurement.

This filter can be found in many varieties such as FKF (Fuzzy Tuned Kalman Filter) [4], Unscented Kalman Filter [5] or classic Kalman Filter.

Regardless of the type of the Kalman filter, there is always the problem of determination of filter settings. The filter defined as follows [6]:

1) Prediction phase
\[
\hat{x}_k^- = A_k \hat{x}_{k-1}^- + B_k u_{k-1} + w_k, \quad (1)
\]
\[
P_k^- = A_k P_{k-1} A_k^T + Q_k, \quad (2)
\]
2) Correction phase
\[
K_k = P_k^- C_k^T (C_k P_k^- C_k^T + R_k)^{-1}, \quad (3)
\]
\[
\hat{x}_k = \hat{x}_k^- + K_k (z_k - (C_k \hat{x}_k^- + v_k)), \quad (4)
\]
\[
P_k = (I - K_k C_k) P_k^-, \quad (5)
\]
where process noise \(w_k\) is assumed to be white gaussian with parameters:
\[
w_k \sim N(0, Q_k), \quad (6)
\]
while measurement noise \(v_k\), which is added to the output equation, has the following parameters:
\[
v_k \sim N(0, R_k) \quad (7)
\]

Settings to find for the presented filter are process noise variance \(Q_k\) and measurement noise variance \(R_k\). Assumptions about the characteristics of the process noise and measurement noise strongly affect the results obtained during estimation of the state vector. The most common problem is the assumption of the additivity and the independency of the process noise variance \(Q\) and the measurement noise variance \(R\).

The first simplification is due to simplified noise model, which allows use of the Kalman filter in applications with complex structure. Further simplification of the Kalman filter, assuming constancy of variance is justified by the immutability of sensors. This means that the filter settings can be set as the variances of the sensors, without the consideration of the influence of physical effects on the overall estimation process. In the literature it very often comes down to the steady state Kalman filter [7]. Here authors decided to create the classic Kalman filter, for which the settings are adjusted after the experiment and the subsequent verification of estimation indicators.

The Kalman filter described by eqs. (1)–(5) can be used for a variety of linear systems for which it is possible to define matrices: \(A\), \(B\) and \(C\). Based on [8] a simple model of the estimator can be derived:
\[
A_k = \begin{bmatrix} 1 & 0 & -dt \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}, \quad (8)
\]
\[
B_k = \begin{bmatrix} dt \\ 1 \\ 0 \end{bmatrix}^T, \quad (9)
\]
\[
C_k = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T, \quad (10)
\]
where \(dt\) stands for the sample time of the integral used to calculate angle from angular velocity.

In addition, it is necessary to define the state vector \(\hat{\mathbf{x}}_k\), input vector \(\mathbf{u}_k\) and measurement vector \(\mathbf{z}_k\):
\[
\hat{\mathbf{x}}_k = \begin{bmatrix} \phi \\ \dot{\phi} \\ b \end{bmatrix}^T, \quad (11)
\]
where \(\phi\) is the angle to be estimated, \(\dot{\phi}\) refers to angular velocity and \(b\) stands for the bias of the gyroscope.

The input vector has only one element and can be defined as:
\[
\mathbf{u}_k = \dot{\phi} \quad (12)
\]
The measurement vector, likewise the input vector, also has only one element:
\[
\mathbf{z}_k = \phi_{acc} \quad (13)
\]

In application, authors used 3-axial accelerometer and 3-axial gyroscope in one chassis (ADIS16405 from Analog Devices). Since the manipulator can be slightly moved in all three directions (\(X\), \(Y\) and \(Z\)), angular velocity from the gyroscope and angle calculated from the accelerometer have to be transformed into the global coordinate system.

Starting from the gyroscope, it can be written that gyroscope measurement vector is equal to:
\[
\Omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + C_X(\phi) \begin{bmatrix} 0 \\ \theta \\ 0 \end{bmatrix} + C_X(\phi)C_Y(\theta) \begin{bmatrix} 0 \\ 0 \\ \psi \end{bmatrix}, \quad (14)
\]
which then may be written in a matrix form:
\[
\Omega = P^S_B \cdot \hat{\Theta}, \quad (15)
\]
where:
\[
P^S_B = \begin{bmatrix} 1 & 0 & -\sin \theta \\ 0 & \cos \phi & \sin \phi \cos \theta \\ 0 & -\sin \phi & \cos \phi \cos \theta \end{bmatrix}, \quad (16)
\]
The next step is to calculate the inverse of the \(P^S_B\) matrix:
\[
P^S_B^{-1} = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \tan \phi \cos \phi / \cos \theta \end{bmatrix}, \quad (17)
\]
Finally, having already the \(P^S_B\) matrix, which transforms raw angular velocities to the global coordinate system, Euler rates may be derived:
\[
\hat{\Theta} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = P^S_B^{-1} \cdot \Omega, \quad (18)
\]
From eq. (18), term \( \dot{\phi} \) is taken directly to the input vector \( \mathbf{u}_k \) in eq. (12).

Similarly to Euler rates, also transformation for the accelerometer data must be done. In order to calculate Euler angles from the accelerations, one can use the general equations for the gravity vector in local coordinate system:

\[
F_\gamma = \begin{bmatrix}
a_x \\
a_y \\
a_z
\end{bmatrix} = R_B^S(\Theta) \cdot \begin{bmatrix}
0 \\
0 \\
g
\end{bmatrix} = \begin{bmatrix}
-\sin \theta g \\
\sin \phi \cos \theta g \\
\cos \phi \cos \theta g
\end{bmatrix},
\]

where \( R_B^S(\Theta) \) is a transformation matrix (from global to local coordinate system):

\[
R_B^S(\Theta) = C_X(\phi) \cdot C_Y(\theta) \cdot C_Z(\psi) =
\begin{bmatrix}
c\phi c\psi & s\phi c\psi & s\psi \\
c\phi s\psi & -s\phi c\psi & c\psi \\
s\phi s\psi & c\phi c\psi & c\phi s\psi
\end{bmatrix}.
\]

From eq. (19), the general form of the acceleration vector in local coordinate system:

\[
\ddot{\mathbf{x}}_g = \begin{bmatrix}
0 \\
0 \\
-\sin \theta
\end{bmatrix}.
\]

Finally, based on \( F_\gamma \), it can be written that:

\[
\phi = \arctan(-\frac{a_y}{a_x})
\]

Angle from (22) goes to the vector \( \hat{\mathbf{a}}_x \). During the estimation process, state vector is updated twice per one measurement period \( dt \). At first, prediction is available in \( \hat{\mathbf{a}}_x \), after that, corrections are made directly on \( \hat{\mathbf{a}}_x \).

### 3. The experiment

#### 3.1. Description of the experiment

During the experiment, data was simultaneously collected from the encoder and the IMU system, with different Kalman Filter settings. For this purpose, the IMU sensor with the data processing system were mounted on the robot’s arm (fig. 2).

The measurement cycle consists of few stages. At first, data is collected at the IMU and sent to the STM processor via SPI interface. Next, the data is transferred to a PC via RS232. On the PC side, C++ application, created in Microsoft Visual Studio (VS), receives the data and estimates the Euler angles. Additionally, on PC, working Automation Studio (AS) receives data from the encoder and displays it in the main window of AS. AS also catches the estimated angle from VS via PVI (the tool for transferring the data). Finally, the angle obtained from the encoder (\( \phi \)) and its estimate are saved to a file.

Overall communication process was presented in the fig. 3.

![Fig. 3. Process of data gathering – a scheme](image)

**Rys. 3. Proces zbierania danych – schemat**

### 3.2. Phases and process of the experiment

During the first phase of the experiment, the IMU was placed on the robot’s arm at a distance of 55 cm from the joint, while in the second phase, it was mounted at a distance of 30 cm from the joint. The phases are illustrated in fig. 4.

![Fig. 4. Phases of the experiment](image)

**Rys. 4. Fazy eksperymentu**

The experiment was carried out, taking into account only the first movement of the joint. This movement was executed with trapezoidal shape of the path.

### 4. Results

As previously stated, during the experiment, settings such as the Kalman filter process noise variance and the variance of the measurement noise (respectively \( Q_k \) and \( R_k \)) were subjected to change. In the first phase, when the sensor was located at the distance of 55 cm from the joint, the following responses from the estimator and the encoder were recorded:

- \( Q_k = 0.01, R_k = 100 \), fig. 5,
- \( Q_k = 0.2, R_k = 10 \), fig. 6,
- \( Q_k = 4, R_k = 10 \), fig. 7,
- \( Q_k = 100, R_k = 10 \), fig. 8,

As it can be seen, one may assess two parameters such as smoothness of the time-course and its fitness. There is a trade-off between these two features. At the time when the value of the noise process variance is increased, the filter reduces the confidence in the data from the gyro for the data from the accelerometer. In this case, significant
is only the ratio of these variances (i.e., $Q_k$ and $R_k$). The authors also performed tests in the second phase, during which, the sensor was mounted 25 cm closer to the joint. For the example settings, the results were shown in the fgs. 9, 10, 11 and 12.

It can be seen that the shorter distance results in decrease of the maximum error of the estimator, which is clearly visible by comparing the plots in fgs. 7 and 11.

In the next step, it was decided to compare all the results of collected data by introducing two quality estimation indicators. The first one is a ratio of the maximum value of the estimation error, which can be defined as:

$$
\epsilon_{\text{max}} = \max(\epsilon_k),
$$

(23)
where $e_k$ is an estimation error in a sample $k$ calculated as a difference between $\phi$ estimated and calculated from the encoder:

$$e_k = \hat{\phi}_k - \phi_k$$ (24)

Another indicator is the sum of the square error, which can be expressed as follows:

$$e_{\text{sum}} = \sum_{k=0}^{k=n} e_k^2,$$ (25)

where $n$ is the estimation horizon.

In tab. 1 all the indicators for the collected results were shown.

**Tab. 1. Estimation quality indicators**

<table>
<thead>
<tr>
<th>No.</th>
<th>$Q_k$</th>
<th>$R_k$</th>
<th>$e_{\text{max}}$ ($^\circ$)</th>
<th>$e_{\text{max}}$</th>
<th>$e_{\text{sum}}$ ($^\circ$)</th>
<th>$e_{\text{sum}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.01</td>
<td>100</td>
<td>19.63</td>
<td>22.76</td>
<td>9970.30</td>
<td>1175476</td>
</tr>
<tr>
<td>2.</td>
<td>0.1</td>
<td>100</td>
<td>12.24</td>
<td>13.53</td>
<td>2771.34</td>
<td>302196</td>
</tr>
<tr>
<td>3.</td>
<td>0.2</td>
<td>10</td>
<td>6.03</td>
<td>4.81</td>
<td>42812.12</td>
<td>44691.9</td>
</tr>
<tr>
<td>4.</td>
<td>0.5</td>
<td>10</td>
<td>3.35</td>
<td>6.01</td>
<td>33625</td>
<td>33012.2</td>
</tr>
<tr>
<td>5.</td>
<td>1</td>
<td>10</td>
<td>11.39</td>
<td>6.52</td>
<td>40446</td>
<td>21530.8</td>
</tr>
<tr>
<td>6.</td>
<td>4</td>
<td>10</td>
<td>12.39</td>
<td>10.76</td>
<td>34772.5</td>
<td>25224.7</td>
</tr>
<tr>
<td>7.</td>
<td>10</td>
<td>10</td>
<td>21.76</td>
<td>11.48</td>
<td>57806</td>
<td>25978.2</td>
</tr>
<tr>
<td>8.</td>
<td>100</td>
<td>10</td>
<td>32.12</td>
<td>14.16</td>
<td>94094</td>
<td>37130.0</td>
</tr>
</tbody>
</table>

In the results, regularity of growth of the integral while reducing the error variance $Q_k$ or increasing the variance $R_k$ can be seen. A similar situation is observed in the case of the maximum error, which is the smallest for the set $Q_k = 0.5$, $R_k = 10$, but it also strongly increases with the distance from the sensor to the joint. Regardless of the quality indicators, one will find that when comparing estimates, it is also necessary to assess the rate of changes, that is, the transmission of vibrations, which does not occur when the process noise variance $Q_k$ is reduced to a certain level.

## 5. Summary and conclusions

In the carried out experiment, authors have focused on a comparison of the results from the estimator and the encoder. The comparison was made for different settings of the filter and for different locations of IMU mounting on the object of research. The results helped to determine that the best settings are near $Q_k = 1$ and $R_k = 10$. Additionally, it was confirmed that the closer to arm’s joint the IMU is fixed, the better quality indicators can be achieved, particularly reducing the maximum error. The whole research was designed to verify operation of the Kalman Filter algorithm in estimation of the tilt angle in the applications with hydraulic drive.

The study confirmed possibilities of application of this type of sensory systems in manipulator with hydraulic drive. In the future, authors plan to enhance the system by introducing the low-pass filter for signals from the accelerometer along with automatic control of the Kalman Filter settings.

## Acknowledgements

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## References


porównanie estymaty kąta odchylenia ramienia robota obliczonej z jednostki IMU oraz enkodera. W pracy zweryfikowano wpływ położenia czujnika IMU na jakość estymaty. Sprawdzeniu poddane zostały również nastawy algorytmu estymującego kąt odchylenia. Ostatecznie zweryfikowano działanie wybranego algorytmu estymującego kąt przy różnych wychyleniach ruchu manipulatora. Celem pracy było sprawdzenie metod estymujących kąt przy użyciu IMU w układach z napędem hydraulicznym.

**Słowa kluczowe:** filtr Kalmana, inercyjna jednostka pomiarowa, manipulator elektrohydrauliczny, enkoder

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